NUMERICAL ANALYSIS OF CHARACTERISTIC MODES ON THE CHASSIS OF MOBILE PHONES

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ABSTRACT

This paper explores the application of the concept of characteristic modes on the chassis of a mobile phone to purposeful design of its radiation characteristics. Focus is on the resonant chassis modes which dominate the overall radiation properties. Modal radiation quality factors are derived from characteristic mode eigenvalues. Antenna–chassis coupling, previously only considered in terms of equivalent circuits, is discussed from a field theoretic point of view. A numerical approach for characteristic mode analysis is presented which is an eigenmode solver extension to the well known NEC2 code. Numerical results are given for wire grid models of a bar-type and a folder-type phone chassis.

Key words: mobile antenna, characteristic mode.

1. INTRODUCTION

Many investigations on antenna design for handheld devices deal with the interaction between the nominal antenna element (coupler) and the chassis on which it is mounted. They have revealed that the radiation performance, especially the bandwidth, largely depends on the coupling between both entities [1]. Less attention has been given to the investigation of the radiation properties of the chassis itself. The influence of the chassis length on the bandwidth of bar-type phones at 900 MHz has been investigated in [2]. A further step was made in [3, 4] which for the first time addressed the analysis of the chassis itself in terms of its characteristic modes [5]. A simple, but limited in scope, approach to the evaluation of resonant chassis modes was described in [6] and examples for purposeful tuning of chassis modes were given there.

The theory of characteristic modes for conducting bodies [5] is in fact a powerful analytical concept for the design of small antennas, similar to what modal analysis means for the design of waveguide circuits. Knowledge of the (typically few) modes which can appreciably be excited at a given frequency gives valuable insight for the placement and design of antenna elements. Antenna–chassis coupling, which has so far mainly been discussed in terms of equivalent circuits [1] can be treated on a field theoretic level in terms of expansion into chassis modes. A first step into this direction is made in the present contribution. Furthermore, a simple expression for evaluation of modal radiation quality factors is derived. To explore the usefulness of these concepts, a numerical approach for analysis of characteristic modes on the chassis of mobile phones is presented which is an extension of the Numerical Electromagnetic Code (NEC2). It allows, in principle, the solution of characteristic mode eigenvalue problems for arbitrary structures, which can be described in terms of a NEC wire grid or surface patch model.

2. CHARACTERISTIC MODE THEORY

The theory of characteristic modes for conducting bodies was introduced by [7] and further elaborated on in [5]. It is based on the properties of the operator \( \hat{Z} \) which maps a surface current density \( \hat{J} \) on the surface \( S \) of a conducting body to the tangential components of the electric field

\[
E_{\text{tan}} = \hat{Z} \hat{J} = (\hat{R} + j \hat{X}) \hat{J},
\]

on \( S \). The operators \( \hat{R} \) and \( \hat{X} \) represent the real and imaginary parts of \( \hat{Z} \), respectively. \( \hat{Z} \) is symmetric from the reciprocity theorem [8] but not Hermitian, whereas \( \hat{R} \) and \( \hat{X} \) are real and symmetric. The problem formulation chosen in [5] therefore leads via the generalized eigenvalue problem

\[
\hat{X} \hat{J}_{n} = \lambda_{n} \hat{R} \hat{J}_{n},
\]

to real eigenvalues \( \lambda_{n} \) and real eigenvectors \( \hat{J}_{n} \). Furthermore, the set of surface current densities \( \{ \hat{J}_{n} : n \in \mathbb{N} \} \) obeys the orthogonality relations

\[
\langle \hat{J}_{m}, \hat{R} \hat{J}_{n} \rangle = 2P_{n} \delta_{mn},
\]

\[
\langle \hat{J}_{m}, \hat{X} \hat{J}_{n} \rangle = 2\lambda_{n} P_{n} \delta_{mn},
\]

\[
\langle \hat{J}_{m}, \hat{Z} \hat{J}_{n} \rangle = 2P_{n}(1 + j \lambda_{n}) \delta_{mn}
\]
The radiation quality factor is a most important quantity for small antennas. Since radiation from a small mobile terminal is typically governed by very few chassis modes, often by a single chassis mode only, the radiation quality factors associated with different (low order) characteristic modes are of high interest.

The common definition of the radiation quality factor $Q_{\text{rad}}$ is

$$Q_{\text{rad}} = \frac{\omega \max(\langle W_{\text{mag}} \rangle, \langle W_{\text{el}} \rangle)}{P_{\text{rad}}}$$

where $\langle W_{\text{mag}} \rangle$ and $\langle W_{\text{el}} \rangle$ are the time averages of the stored magnetic and electric field energies, respectively.

Application of the complex Poynting theorem with respect to the domain $G$, bounded outside by a sphere in the far-field region and inside by the surface $S$ of the conducting chassis, yields

$$\frac{1}{2} \iint_{S} (E \times H^*) \cdot n \, dS = P_{\text{rad}} + 2 j \omega \langle W_{\text{mag}} \rangle - \langle W_{\text{el}} \rangle$$

where, on $S$, the direction of the normal $n$ is chosen to point into $G$. Since on $S$

$$(E \times H^*) \cdot n = (\vec{Z} J_s)(H^* \times n) = J_s^{\ast} (\vec{Z} J_s)$$

one obtains for the $n$-th mode

$$\frac{1}{2} \langle J_{s,n} \rangle (\vec{Z} J_s) = P_{\text{rad}} + 2 j \omega \langle W_{\text{mag}} \rangle - \langle W_{\text{el}} \rangle.$$  \hfill (10)

Comparison with (5) yields $P_{\text{rad}} = P_n$ and

$$\lambda_n = 2 \omega \langle W_{\text{mag},n} \rangle - \langle W_{\text{el},n} \rangle,$$  \hfill (11)

i.e. a zero of the eigenvalue $\lambda_n$ over frequency indicates resonance of the respective mode. The expression (11) for $\lambda_n$ is formally identical with the definition of the so-called phase quality factor, a quantity which, however, is not useful near resonance. Another familiar quality factor definition, based on the steepness of the phase about resonance,

$$Q_{s,n} = \frac{\omega_n}{2} \left| \frac{d\lambda_n}{d\omega} \right|_{\omega=\omega_n}$$

is arrived at by taking the derivative of the eigenvalue with respect to frequency. Although the quality factors calculated according to definitions (7) or (12), respectively, are not identical, a useful interrelation exists. In the present paper only quality factors at resonance, i.e. for $\lambda_n(\omega_n) = 0$, are considered. To evaluate the frequency derivatives of the reactive part of the stored magnetic and electric energies, $\langle W_{\text{mag},n} \rangle$ and $\langle W_{\text{el},n} \rangle$, recourse is taken to equivalent circuit models of series or parallel type, respectively, near resonance. Hints as to a more general treatment will be given below. For a series type resonance the frequency dependence of the reactive parts of $\langle W_{\text{mag},n} \rangle$ and $\langle W_{\text{el},n} \rangle$ near resonance can be included in (11) by multiplying these terms with $\omega/\omega_n$ and $\omega_n/\omega$, respectively, and vice versa for a parallel type resonance. It is then easily verified that at resonance

$$Q_{\text{rad},n} = \frac{1}{2} Q_{s,n} = \frac{\omega_n}{2} \left| \frac{d\lambda_n}{d\omega} \right|_{\omega=\omega_n}. \hfill (13)$$

Numerical values thus obtained are in good agreement with values previously obtained by fitting the first moment of induced current density on a chassis, obtained by simulation under plane wave excitation, to a generic resonator model [6].

A general derivation which does not take recourse to equivalent circuit concepts is possible following the concepts derived in [9] where a complex frequency formulation of the Poynting theorem is used to establish a relation between $\langle W_{\text{mag},n} \rangle = \langle W_{\text{el},n} \rangle$ and its time derivative by exploitation of the Cauchy–Riemann conditions.

4. ANTENNA – CHASSIS COUPLING

The interaction between a mobile phone chassis and the antenna or, more appropriate, the coupler is basic for mobile phone antenna design. Conceptual analysis, in the genuine meaning of the term, i.e. of decomposing a complex problem into simpler pieces, is possible by separately considering the characteristic modes of the chassis and the exciting (or induced) fields at the coupler. A strict distinction between chassis and coupler is of course impossible since the coupler is part of the chassis. Typically, however, the coupler is much smaller than the chassis. For a conceptual understanding one may therefore approximate the surface current density $J_s$ of the chassis as a superposition of the characteristic modes of the unperturbed chassis, i.e. in the form [5]

$$J_s = \sum_n \frac{\langle J_{s,n} \rangle E_{\text{tan}}^{\text{exc}}}{(1 + j \lambda_n) 2 F_n J_{s,n}} \hfill (14)$$

where $E_{\text{tan}}^{\text{exc}}$ denotes an exciting electric field, tangential to $S$, in the present context assumed due to the (external)
coupler. Note that $P_n$ enters into (14) as a normalization constant only. It is immediately obvious from (14) that only modes near resonance can make a significant contribution to the total radiated power since, due to characteristic mode’s pattern orthogonality,

$$\sum_n |\langle J_{s,n}, E_{\text{tan}}^{\text{ex}} \rangle|^2$$

of the short edge leads to minimum distance from the PCB and thereby smallest volume consumption. Moreover, rewriting the $n = 1$ term on the right hand side of (15) at resonance ($\omega = \omega_1$) using the approximation (17) tells that the power radiated from this mode is approximately given by

$$P_{\text{rad},1} \approx \frac{1}{2} |q_1|^{2} = \frac{1}{2} \frac{1}{P_1} R_{\text{rad},1} |i^{\text{ex}}|^{2},$$

i.e. the radiation resistance for the coupler and thereby bandwidth are maximized.

5. NUMERICAL METHOD

The numerical evaluation of characteristic modes amounts to the solution of the generalized real eigenvalue problem (2) under free-space boundary conditions. The obvious choice of a suitable numerical method is the method of moments (MoM) because it avoids any artificial termination of the computational volume (absorbing boundary conditions) which might result in non-physical eigenmodes.

The well known Numerical Electromagnetics Code (NEC2) [10] was chosen as the code basis for present exploratory study. The original code allows for, in principle, arbitrary structures which can be approximated in terms of a wire grid or surface patch model. It solves the inhomogeneous field problem for a given excitation (voltage or current source or incident plane wave). For the present purposes the code was extended to allow also for the solution of the eigenvalue problem (2). The system matrix

$$Z = \langle w_i, \hat{Z} b_j \rangle_{i,j}$$

i.e. the projection of the operator $\hat{Z}$ on the basis functions $b_j$ and weighting functions $w_i$, is generated by the unmodified NEC2 code. Instead of solving an inhomogeneous matrix equation, however, $Z$ is passed to a
LAPACK [11] solver for the real symmetric generalized eigenvalue problem. The eigenvector is passed back to the original NEC2 code to write a regular NEC output file to allow for inspection with one of the widely available post-processing tools. A slightly patched version of XNECVIEW [12] was used for the graphics in this paper.

One aspect of interest is the relation between the solution spaces of the analytical eigenvalue problem (2) and its discrete counterpart. For wire grid structures for instance, NEC makes use of continuous basis functions $b_j$ along wire segments but employs Dirac pulses as weighting functions $w_i$. As a consequence, the matrices $\mathbf{R}$ and $\mathbf{X}$ are not symmetric. Nevertheless, since the matrix $\mathbf{Z}$ may be envisioned as obtained from a symmetric matrix

$$\tilde{\mathbf{Z}} = \tilde{\mathbf{R}} + j \tilde{\mathbf{X}} = (b_k, \tilde{\mathbf{Z}} b_j)_{k,j} \quad (20)$$

which is left multiplied by a non-singular projection matrix

$$\mathbf{P} = (\langle w_i, b_k \rangle)_{i,k} \quad (21)$$

the matrices $\tilde{\mathbf{R}}$ and $\tilde{\mathbf{X}}$ are subject to the same non-singular transformation and the properties of a generalized real symmetric eigenvalue are retained. Numerical experiments have confirmed that all eigenvalues of interest, i.e. those of small or moderate magnitude, are obtained real.

6. APPLICATION EXAMPLES

6.1. Bar Type phone structure

The wire grid structure in Fig. 2 is a rudimentary model for a bar type phone chassis. While the battery and possible conductive parts of the housing are ignored it is a fair approximation of the PCB. Dimensions are 100 mm × 40 mm. Fig. 2b shows the frequency dependence of the eigenvalues $\lambda_n$ with magnitude less than 10 in the frequency range from 1 GHz to 3.2 GHz. The graphs are constructed based on the correlation between eigenvectors at adjacent frequency samples. According to the discussion in Section 3, characteristic mode resonances are of highest interest. Entries (A–D) in Table 1 show resonance frequencies and radiation quality factors at resonance after (13). The associated current patterns are shown in Fig. 3. XNECVIEW encodes current magnitude on a wire by line thickness and phase by color. Here blue and red correspond to positive and negative signs. Inspection of the magnitudes of the first and second higher order eigenvalues at the half-wave resonance ($\lambda_2$ and $\lambda_3$ in Table 1) confirms the dominance of the major axis half-wavelength resonant mode (A) at lower frequencies. The denominator for $n = 2$ on the right hand side of (15) is already by a factor 100 larger. It is interesting to note that the low order eigensolutions change only very slowly with frequency and may in fact be considered almost constant along each graph in Fig. 2b. Relevant chassis currents in the given frequency range are therefore essentially superpositions of the major axis and minor axis electric dipole modes (A) and (C), the quadrupole mode (B) and the magnetic dipole mode

![Figure 2](a) Wire grid model of a 100 mm × 40 mm plate and (b) frequency dependence of the first few eigenvalues for this structure.

![Table 1](The first 4 characteristic mode resonances for the plate structure after Fig. 2a (A–D), the closed folder type structure after Fig. 4a (E–H) and the open folder type structure after Fig. 6a (I–L) together with the two next higher eigenvalues at the same frequency.)

<table>
<thead>
<tr>
<th>label</th>
<th>$f_1 / \text{MHz}$</th>
<th>$Q_{\text{rad},1}$</th>
<th>$\lambda_2$</th>
<th>$\lambda_3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>(A)</td>
<td>1260</td>
<td>2.3</td>
<td>-10.01</td>
<td>20.84</td>
</tr>
<tr>
<td>(B)</td>
<td>2679</td>
<td>3.0</td>
<td>-0.11</td>
<td>0.87</td>
</tr>
<tr>
<td>(C)</td>
<td>2739</td>
<td>2.5</td>
<td>0.13</td>
<td>-0.70</td>
</tr>
<tr>
<td>(D)</td>
<td>3081</td>
<td>2.3</td>
<td>0.53</td>
<td>0.57</td>
</tr>
<tr>
<td>(E)</td>
<td>993</td>
<td>23.5</td>
<td>-11.38</td>
<td>-16.12</td>
</tr>
<tr>
<td>(F)</td>
<td>1957</td>
<td>1.1</td>
<td>-1.76</td>
<td>6.91</td>
</tr>
<tr>
<td>(G)</td>
<td>2761</td>
<td>11.1</td>
<td>0.36</td>
<td>-0.57</td>
</tr>
<tr>
<td>(H)</td>
<td>3049</td>
<td>2.4</td>
<td>-0.27</td>
<td>-0.74</td>
</tr>
<tr>
<td>(I)</td>
<td>982</td>
<td>3.3</td>
<td>-17.76</td>
<td>37.57</td>
</tr>
<tr>
<td>(J)</td>
<td>1985</td>
<td>4.0</td>
<td>1.29</td>
<td>-2.44</td>
</tr>
<tr>
<td>(K)</td>
<td>2926</td>
<td>2.5</td>
<td>-0.26</td>
<td>-0.62</td>
</tr>
<tr>
<td>(L)</td>
<td>3072</td>
<td>5.1</td>
<td>-0.05</td>
<td>0.23</td>
</tr>
</tbody>
</table>
The availability of three orthogonal modes (patterns) with eigenvalues of magnitude less than or around 2 in the 2.4 GHz ISM band is an interesting observation for the design of MIMO antenna systems on mobile phones.

6.2. Folder Type phone structure

Simple wire grid models of a closed and open folder type phone chassis and the first few eigenvalues over frequency are shown in Figs. 4 and 6, respectively. The base part is modeled as a 70 mm × 40 mm plate and the flip part as a 50 mm × 40 mm plate at 15 mm distance. The first four resonances (E–H) and (I–L) are again tabulated in Table 1. The associated current patterns for the closed structure are given in Fig. 5. The first resonance (E) is easily identified as that of a shorted quarter-wave parallel plate resonator which explains its high radiation quality factor. The electrical dipole mode resonance (F) has a remarkably low quality factor, indicating that this structure permits large bandwidth over the DCS, PCS and UMTS bands. The transverse half-wave resonance (G) again yields poor radiation, as indicated by its high quality factor, due to counter-oriented currents on base and flip part.

Figure 4. (a) Wire grid model of a closed folder type phone structure and (b) frequency dependence of the first few eigenvalues for this structure.

7. CONCLUSION

The theory of characteristic modes for conducting bodies was applied to study the radiation properties a mobile phone chassis. An expression for modal radiation quality factors in terms of characteristic mode eigenvalues was given. An approximate model for the coupling between a resonant chassis mode and a capacitive coupling element (“antenna”) was derived considering the major axis half-wave resonance of a bar type phone chassis. Numerical results for eigenvalues over frequency, modal current patterns and radiation quality factors were presented for rudimentary wire grid models of a bar and a folder type mobile phone chassis. The approach is found to provide useful insight into the radiation properties of a chassis. It is expected to be particularly useful for the design of multiple antennas on small platforms for future MIMO systems.

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Figure 5. Current patterns for the characteristic mode resonances (E–H) given in Table 1 (closed folder structure after Fig. 4a).

Figure 6. (a) Wire grid model of an open folder type phone structure and (b) frequency dependence of the first few eigenvalues for this structure.