FIR-filter based equalization of ultra wideband mutual coupling on linear antenna arrays

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Abstract—This paper describes a compensation method of ultra wideband mutual coupling using finite impulse response (FIR)-filters to equalize the frequency characteristic of antenna interactions. The result is a compensated far-field impulse response, which yields an radiated electric field similar to one caused by an ideal uncoupled antenna array. On the basis of an antenna array impulse response, which includes antenna interaction, a mathematical framework is derived to configure the FIR-filters. Finally, simulations are presented, where the preprocessed radiated electric field of an antenna array coincides with the electric field caused by a single isolated antenna without any coupling effects.

I. INTRODUCTION

Array processing and smart antenna concepts have been recently proposed for radar and communication systems. They offer a promising solution to enhance gain and to reduce interference with spurious signals and multipath components. It is evident, that these advantages can directly be assigned to ultra wideband (UWB) communication systems.

A concept for UWB array processing has already been presented considering realistic antenna and channel properties [1]. However, one effect of antenna arrays is often neglected, even for narrowband applications but even more than ever for UWB applications: The interaction and mutual effects among the antenna elements in the array, usually denoted as mutual coupling. An obstacle or just another antenna can alter the surface current distribution of the antenna under test. Even if an antenna is not fed by its own, there can be an induced surface current by radiation from a neighboring antenna, which leads to secondary radiation. So, depending on the interelement distance, mutual coupling can affect the radiated or transmitted signal significantly.

In narrowband systems mutual coupling is usually described by an impedance matrix, containing the self- and mutual impedance of an antenna, permitting the description of antenna mismatch [2].

In UWB short pulse systems, antennas are described by other parameter, e.g. by their time-domain impulse response function [3]. This angular-temporal function yields a distortion of the signal, transmitted or received by the antenna. Hence, if this function can be derived including mutual coupling effects, we can describe how the pulse shape alters due to antenna interaction [4].

In [1] apart from beamforming, we used analogue finite impulse response (FIR)-filters at RF to equalize antenna and channel characteristics. In this paper we extend the usage of FIR-filters in view of UWB mutual coupling compensation. Analogue FIR-filters are very suitable for microwave applications, since they can be realized with MIC technology up to a few gigahertz and are suitable for monolithic integration.

In Section II we show the principle of calculating the impulse response of antenna and antenna array, respectively. A mathematical framework is presented, where antenna mutual coupling effects are included in the antenna array impulse response.

In Section III an FIR-filter based mutual coupling compensation concept is presented. Finally, some simulation results are given as a “proof-of-concept”.

II. TIME-DOMAIN ANTENNA CHARACTERIZATION

A. Single antenna-element impulse response

The electric far field at an observation point \( r \) can be calculated by

\[
E(r, t) = -\frac{\partial}{\partial t} A(r, t). \tag{1}
\]

\( A \) is the farfield radiating vector potential, given by [5]

\[
A(r, t) \simeq \frac{\mu}{4\pi R} \int J \left( r', t - \frac{r - \hat{r} \cdot r'}{c_0} \right) dV, \tag{2}
\]

where \( c_0 \) is the speed of light, \( r' \) is the source coordinate, \( \hat{r} \) is the unity coordinate of the observation point, \( R = |r| \) is the distance between the observation point and the antenna and \( J \) is the surface current distribution on the antenna. Considering that the antenna is fed by a Dirac-impulse \( I^+(t) = \delta(t) \) one can describe the time-domain antenna impulse response by

\[
h_0(\hat{r}, t) = \int J_\delta \left( r', t - \frac{r - \hat{r} \cdot r'}{c_0} \right) dV. \tag{3}
\]

Substitution of Eqs. 3 and 2 into Eq. 1 yields [6]
h(t) (m/ns)

**B. Antenna array with single fed antenna**

If a single fed antenna is placed in an array of antennas (see Fig. 1), the surface current changes due to the presence of additional antennas in close vicinity. Furthermore, the fed antenna induces surface currents on the other antenna elements of additional antennas in close vicinity. Hence, one can determine an impulse response of the antenna array at an observation angle \( \varphi \) and interelement distance \( d \).

![Fig. 1. Mutual coupling effects of a two-element array with one element fed by a pulse source and the other element terminated. Note: The observation point is plotted closer here than defined by the far zone rule.](image1)

The variation of the impulse response of an antenna array compared to the impulse response of a single isolated antenna can be observed in Fig. 2, where \( N = 3 \) antennas were considered with an interelement distance of \( d = 10 \text{ mm} \).

It can be seen that antenna interaction yields a distortion of the impulse response. In order to quantify the variation of the antenna array impulse response from the impulse response of a single isolated antenna, we investigate their correlation as a function of the observation angle \( \varphi \).

A correlation coefficient of \( r(\varphi) = 1 \) means that the two signals are identical, \( r(\varphi) = 0 \) expresses no correlation.

In Fig. 3 we show the correlation of a \( N = 2 \) and \( N = 3 \) antenna array as a function of the observation angle \( \varphi \) from \( 0^\circ \) to \( 90^\circ \) and the interelement distance \( d \). The observation point \( r \) is now 2-dimensionally expressed by the observation angle \( \varphi \) around the antenna array. It can be seen that the array impulse response highly depends on the observation angle and the interelement distance but also on the number of antenna elements. Another parameter, not shown here, is of course the antenna type. This simulation was performed on the basis of a Vivaldi antenna [7].

Using this graph it is possible to quantify the correlation between the curves shown in Fig. 2, which is due to the dashed blue curve in Fig. 3 approx. \( r(\varphi) = 0.83 \).

**C. Antenna array with \( N \) fed antennas**

In classical narrowband antenna array synthesis, each antenna is fed by individual currents \( I_n \) which, due to the synthesis algorithm, are appropriately weighted and phase shifted [2]. The phase shift mainly yields a steering of the main beam of the radiation pattern. For ultra broadband signals the main beam of the radiation pattern can be controlled by time-delays in the simplest case. But even more complex functions, which can be realized by filters, are conceivable, as we have shown in [1].

The currents \( I_n \) are realized by splitting a current \( I_0^+ \) into \( N \) branches and weighting them. Hence, if each antenna element is weighted by a function \( w_n(\hat{r}, t) \), the overall antenna
array impulse response considering mutual coupling can be expressed by

\[
h(\hat{r}, t) = \sum_{n=1}^{N} w_n(\hat{r}, t) \ast h_n(\hat{r}, t). \tag{8}
\]

### III. Compensation of Antenna Coupling

It has been shown that antenna mutual coupling yields a distortion of the antenna impulse response (see Fig. 2). If the distortion is well behaved (e.g., no nulls in the frequency spectrum), it is possible to compensate the distortion using appropriate filter techniques, e.g., FIR-filters. This technique compensates the far-field impulse response by preprocessing the feeding currents of the antennas. Hence, the electric field seems to be radiated by an “ideal” uncoupled antenna array. The mutual coupling caused network mismatch will not be compensated by the FIR-filters.

There are two conceivable setups for placing filter elements in the system:

1) Behind each antenna element in the array a filter-element can be placed to compensate the array impulse responses separately, Fig. 4(a).
2) A single filter is placed between the feeding source and the signal splitter, Fig. 4(b).

The former solution is independent from the weighting functions \( w_n(\hat{r}, t) \) but the FIR-filter functions need to be calculated for each antenna separately. This increases the complexity of the compensation. On the other hand it is simple to combine the weighting functions \( w_n(\hat{r}, t) \) with the decoupling functions \( h_{\text{FIR,n}}(\hat{r}, t) \) at each antenna, creating one filter matrix by calculation an overall filter functions behind each antenna elements like

\[
\tilde{h}_{\text{FIR,n}}(\hat{r}, t) = h_{\text{FIR,n}}(\hat{r}, t) \ast w_n(\hat{r}, t). \tag{9}
\]

The advantage of the second concept is that only one decoupling function for the overall antenna array has to be determined. Unfortunately, \( h_{\text{FIR}}(\hat{r}, t) \) is a function of the weighting functions \( w_n(\hat{r}, t) \). Hence, for each change of the weighting functions, e.g. if the main beam is pointed to another direction, the FIR-filter function has to be recalculated. However, also this concept of a single decoupling filter can be combined with the weighting functions to a single filter matrix. A framework to a similar problem has been presented [8].

The special case that only one antenna element is fed and the other ones are passive (see Sec. II-B) can be considered by setting the weighting function \( w_n(\hat{r}, t) = 1 \) for the fed antenna and \( w_n(\hat{r}, t) = 0 \) for all passive antennas. For that case both concepts being discussed are identical.

Which concept generally should be chosen depends on the application and does not yield a clear answer. In the following section, the concept of a single decoupling filter will be further discussed.
A. FIR-filter concept

Fig. 5 shows the structure of an FIR-filter, placed between the feeding source and the signal splitter. The input signal $I_0^+(t)$ is alternately delayed by incremental time delays $\tau$ and multiplied by real weighting coefficients $a_m$, where $-1 \leq a_m \leq 1$. The time-domain impulse response follows

$$h_{\text{FIR}}(t) = \sum_{m=1}^{M} a_m \delta(t-(m-1)\tau), \quad (10)$$

where the weighting coefficients $a_m$ are adjustable and have to be appropriately configured. The incremental time delay $\tau$ is generally a function of the highest frequency in the spectrum and is not variable.

The overall impulse response of the antenna array and the FIR-filter yields

$$h_{\text{arr}}(\vec{r}, t) = h_{\text{FIR}}(\vec{r}, t) \ast h(\vec{r}, t), \quad (11)$$

Now we determine $h_{\text{FIR}}(\vec{r}, t)$ such that $h_{\text{arr}}(\vec{r}, t)$ is equal to the ideal reference function $h_{\text{ref}}(\vec{r}, t)$

$$h_{\text{FIR}}(\vec{r}, t) = h_{\text{ref}}(\vec{r}, t) \ast h(\vec{r}, t) = h_{\text{ref}}(\vec{r}, t) \quad (12)$$

where

$$h_{\text{ref}}(\vec{r}, t) = h_0(\vec{r}, t) \quad (13)$$

$$\ast \left( \sum_{n=1}^{N} w_n(\vec{r}, t) \ast \delta \left( t-(n-1)\frac{d}{c_0}\sin\varphi \right) \right) \quad (14)$$

is the convolution of the impulse response of a single isolated antenna with the array factor.

After converting Eq. 12 and Eq. 13 into the frequency domain, one can solve the expressions for the FIR frequency response like

$$H_{\text{FIR}}(\vec{r}, \omega) = \frac{H_{\text{ref}}(\vec{r}, \omega)}{H(\vec{r}, \omega)}, \quad (15)$$

where $H_{\text{FIR}}(\vec{r}, \omega)$ is the frequency response of the FIR-filter, $H_{\text{ref}}(\vec{r}, \omega)$ is the frequency response of the ideal uncoupled array factor and $H(\vec{r}, \omega)$ is the frequency response of the coupled antenna array.

The filter coefficients $a_m$ can be calculated directly by Eq. 15 or numerically, e.g. using optimization algorithms like

$$\min \left( \max |H(\vec{r}, \omega) \cdot (A \cdot a_m) - H_{\text{ref}}(\vec{r}, \omega)| \right), \quad (16)$$

where the term $A \cdot a_m$ yields the frequency-domain FIR-filter transfer function.

The improvement of the signal quality due the FIR-filter based antenna mutual coupling compensation is shown in Fig. 6 in terms of the correlation (compare Fig. 3) for $N = 2$ and $N = 3$ antenna elements with an interelement distance of $d = 10\,\text{mm}$. Inside the angle interval from $0^\circ$ to $40^\circ$ the diagram indicates approx. ideal correlation close to 1. Then the correlation decays marginally and some minima occur. This happens, if the frequency response of the coupled antenna array $H(\vec{r}, \omega)$ exhibit nulls such that the denominator in Eq. 15 becomes zero for that frequency and the whole expression does not yield a finite value. This effect can be considered as a limit of FIR-filter based mutual coupling compensation.

B. Simulation results

1) Single fed antenna: Fig. 7(a) shows different simulated time-domain electric fields in the far zone at a distance $R = 250\,\text{mm}$ under an observation angle $\varphi = 45^\circ$ of a single fed antenna. The component $E_{\text{arr}}(t)$ is caused by a single isolated Vivaldi antenna [7], which is fed by a gaussian pulse (bandwidth $f_l - f_h = 0.1 \,\text{GHz} - 12 \,\text{GHz}$). One can see that the pulse shape of the electric field changes, if a second antenna is placed with an inter-element distance of $10\,\text{mm}$ next to the first antenna, denoted by $E_{\text{arr}}(t)$. Using the corresponding impulse response function, a 31-elements FIR-filter with $\tau = 1/(2f_h) \approx 42 \,\text{ps}$ is configured to compensate the mutual effect and is placed between the source and feeding antenna. The electric field $E_{\text{comp}}(t)$ now coincides with the reference field $E_{\text{ref}}(t)$. The correlation factor is $r(\varphi) = 0.98$ after compensation.

2) N equally fed antennas: Fig. 7(b) shows the simulated time-domain electric fields, assuming that all antenna elements of an $N = 3$ array are equally fed by a weighting function $w_n = 1 \forall n$, producing a beam at broadside ($\varphi_0 = 0^\circ$). Also
(a) Two antenna elements, where one antenna is fed and the other one is passive.

(b) $N = 3$ equally fed antennas.

Fig. 7. Electric farfield comparison at $\varphi = 45^\circ$: The reference field $E_{\text{ref}}(t)$ is calculated without any coupling effects, the electric field $E_{\text{arr}}(t)$ is calculated considering mutual coupling and the field $E_{\text{comp}}(t)$ arises, if the antenna array is preprocessed by an appropriately configured FIR-filter.

here, the electric field $E_{\text{comp}}(t)$ coincides with the reference field $E_{\text{ref}}(t)$ and the correlation factor yields $r(\varphi) = 0.98$.

IV. CONCLUSION

In this contribution antenna mutual coupling for UWB short pulse antenna array has been investigated. A mathematical framework was presented describing an antenna array impulse response including coupling effects. This was the basis in order to derive a concept to compensate mutual coupling effects in UWB antenna arrays by FIR-filters. Two proof-of-concept simulations showed that the FIR-filter preprocessing results in an electric field in the far zone, which coincides with the electric field radiated by a single antenna element.

REFERENCES


